

# Paramagnetismo de Pauli

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$$\epsilon(\mathbf{k}, \downarrow) = \frac{\hbar^2 |\mathbf{k}|^2}{2m} + \mu_B B$$

$$\epsilon(\mathbf{k}, \uparrow) = \frac{\hbar^2 |\mathbf{k}|^2}{2m} - \mu_B B.$$

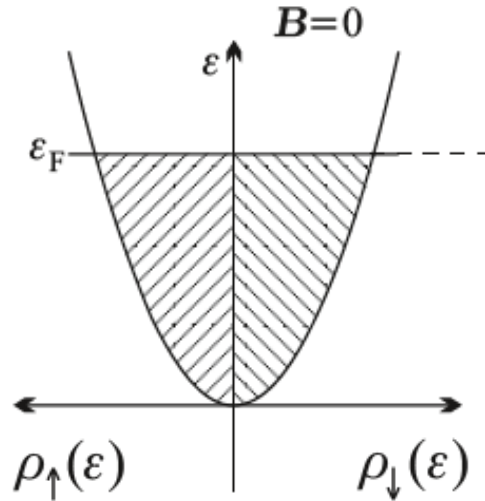
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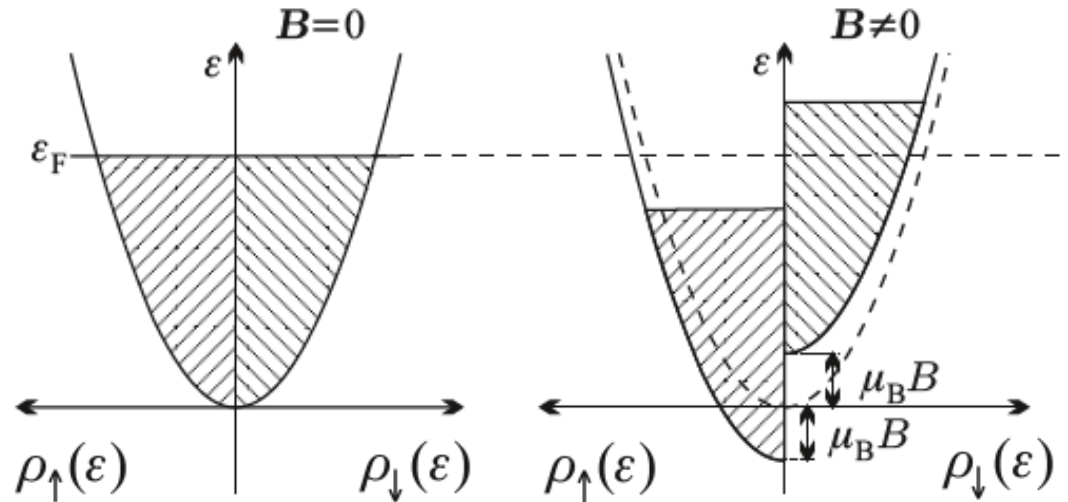


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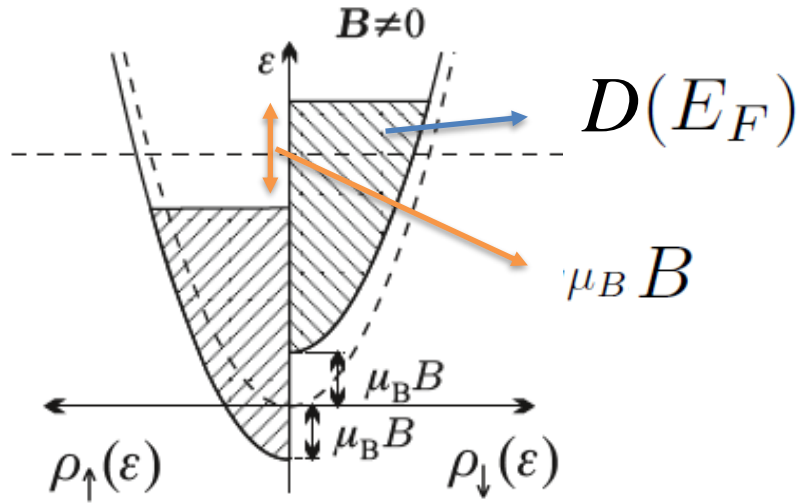
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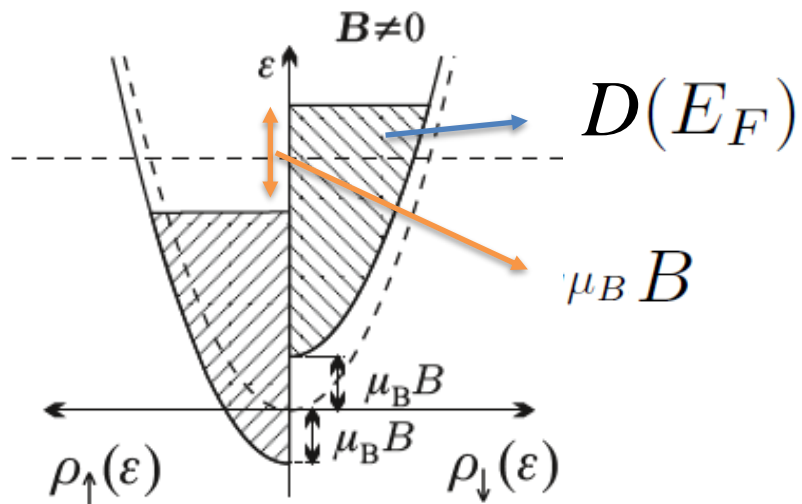
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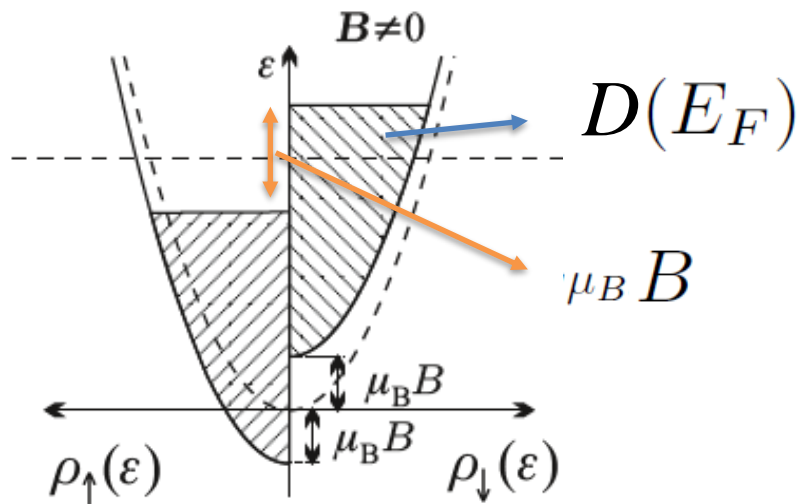


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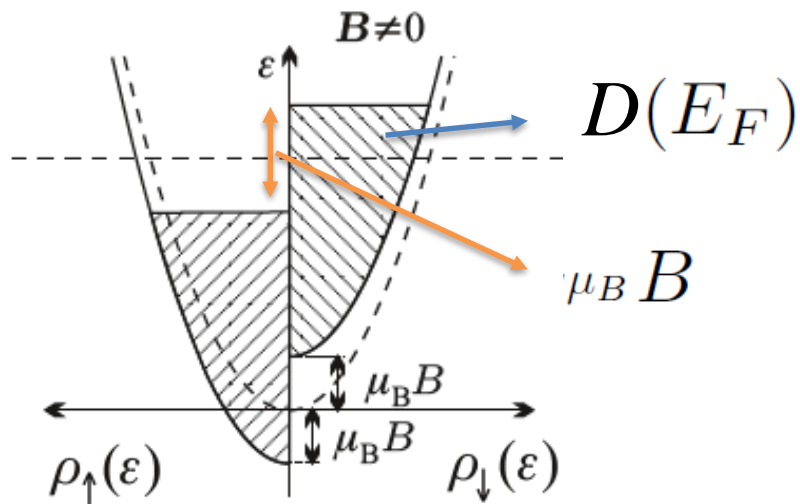


$$\chi_{Pauli} = \text{constante}$$



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$$R_W = \frac{4\pi^2 k_B^2 T}{3\mu_0 (g_e \mu_B)^2} \frac{\chi_P}{c_{el}}$$

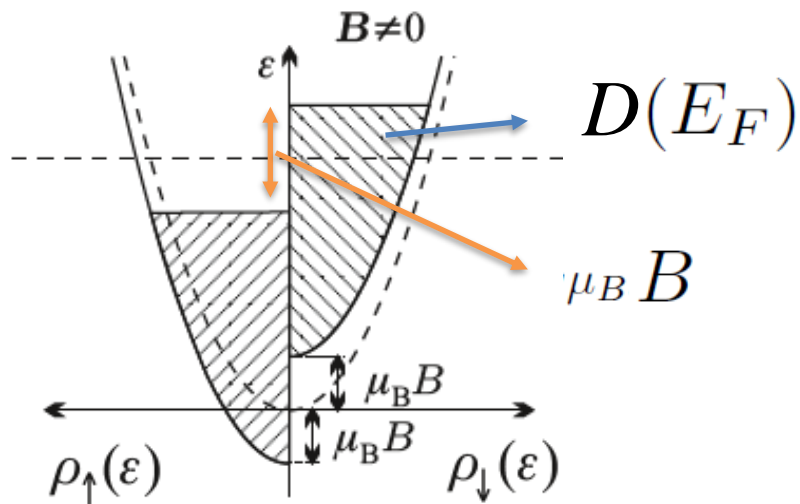
Razão de Wilson

# Paramagnetismo de Pauli

Em baixas temperaturas

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - g\mu_B \mathbf{B} \cdot \boldsymbol{\sigma}$$

$$\chi_p = \frac{3N\mu_B^2}{2\epsilon_F} \left[ 1 - \frac{\pi^2}{12} \left( \frac{T}{T_F} \right)^2 + \dots \right]$$



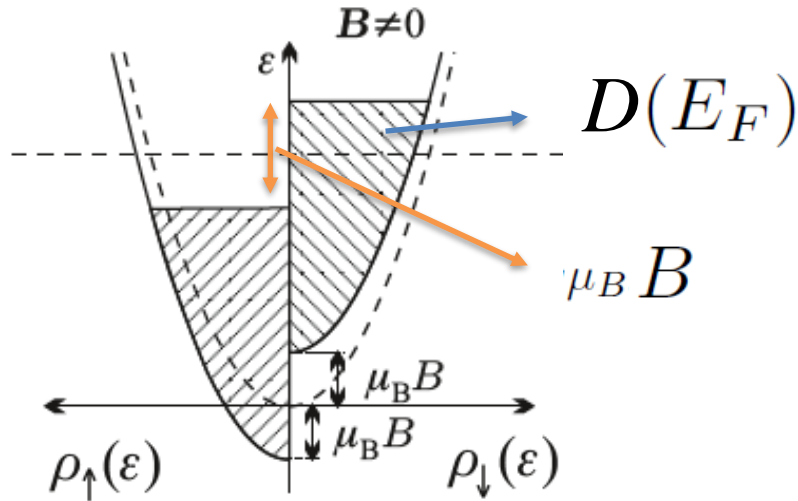
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Em altas temperaturas



$$M = N\mu_B \tanh \left( \frac{\mu_B B}{kT} \right)$$

$$\chi = \mu_0 \frac{dM}{dB} = \mu_0 \frac{N\mu_B^2}{kT}$$

Curie Law

# Diamagnetismo de Landau

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$$\mathcal{H} = \frac{1}{2m} (\vec{p} - q\vec{A})^2$$

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$$\mathbf{B} = \nabla \times \mathbf{A}$$

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Notem que

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Landau Ansatz

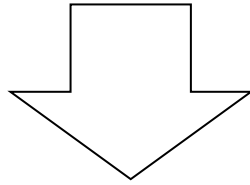
$$\psi(x, y, z) = u(x) e^{ik_y y} e^{ik_z z}$$

# Diamagnetismo de Landau

$$-\frac{\hbar^2}{2m_e} \frac{d^2 u(x)}{dx^2} + \frac{\hbar^2}{2m_e} \left( k_y + \frac{eB}{\hbar} x \right)^2 u(x) + \frac{\hbar^2 k_z^2}{2m_e} u(x) = \varepsilon u(x)$$

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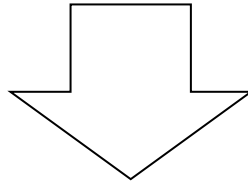
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$$-\frac{\hbar^2}{2m_e} \frac{d^2 u(x)}{dx^2} + \frac{1}{2} m_e \left( \frac{eB}{m_e} \right)^2 \left( x + \frac{\hbar}{eB} k_y \right)^2 u(x) = \left( \varepsilon - \frac{\hbar^2 k_z^2}{2m_e} \right) u(x)$$

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$$\omega_c = \frac{eB}{m_e}$$

$$x_0 = -\frac{\hbar}{eB} k_y$$

Frequência de cíclotron

# Diamagnetismo de Landau

$$-\frac{\hbar^2}{2m_e} \frac{d^2 u(x)}{dx^2} + \frac{1}{2} m_e \omega_c^2 (x - x_0)^2 u(x) = \left( \varepsilon - \frac{\hbar^2 k_z^2}{2m_e} \right) u(x)$$

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Quantum Harmonic oscillator!!


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Quantum Harmonic oscillator!!

Polinômios de Hermite (deslocados de  $x_0$ )

$$u_n(x) = \frac{1}{\pi^{1/4} l_0^{1/2} \sqrt{2^n n!}} H_n \left( \frac{x - x_0}{l_0} \right) e^{-(x-x_0)^2 / 2l_0^2}$$


$$l_0 = \sqrt{\frac{\hbar}{m_e \omega_c}} = \sqrt{\frac{\hbar}{eB}}$$

Comprimento magnético  
(associado à órbitas no espaço real)

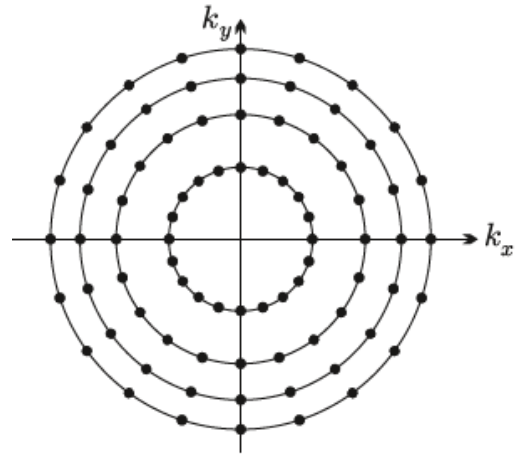


# Diamagnetismo de Landau

$$\varepsilon = \left(n + \frac{1}{2}\right) \hbar \omega_c + \frac{\hbar^2 k_z^2}{2m_e},$$

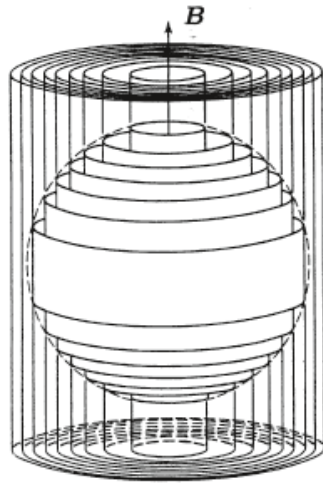
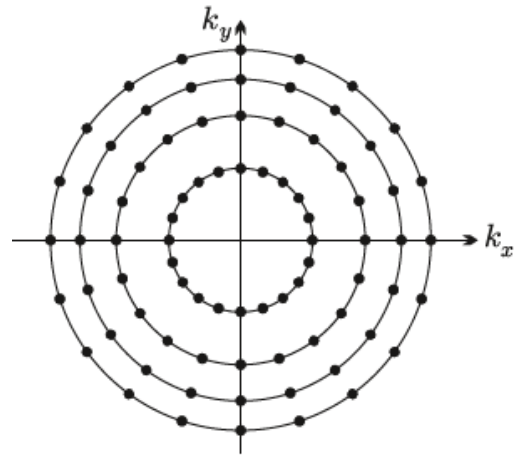
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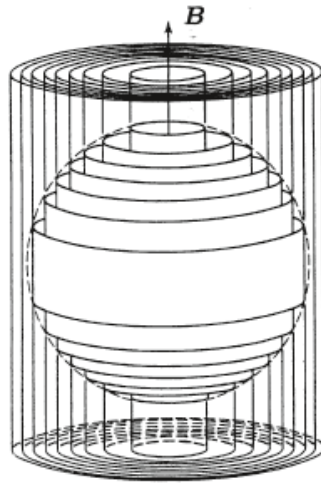
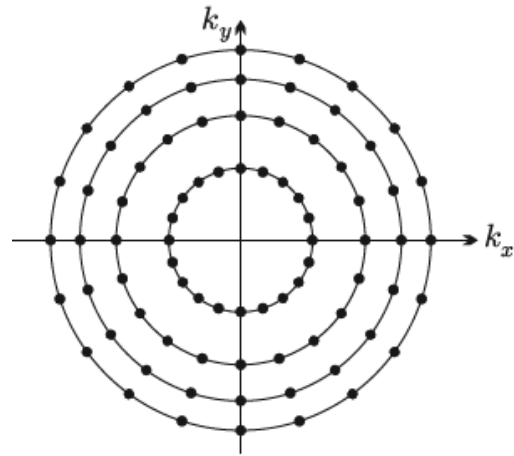
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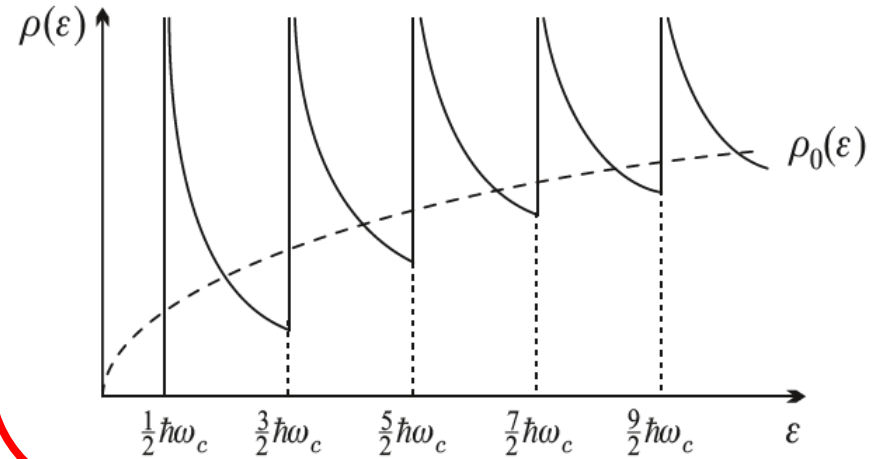


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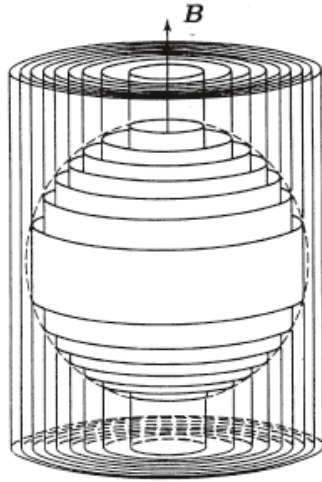
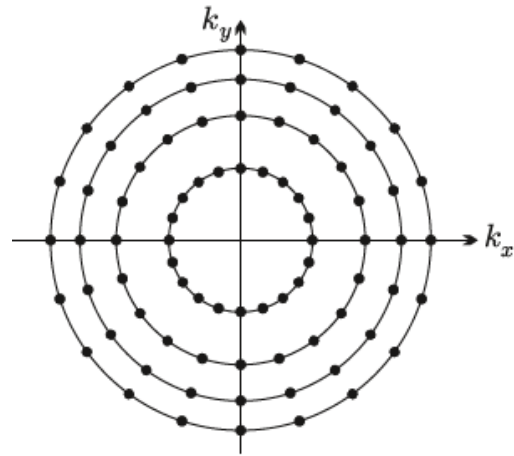


## Density of States

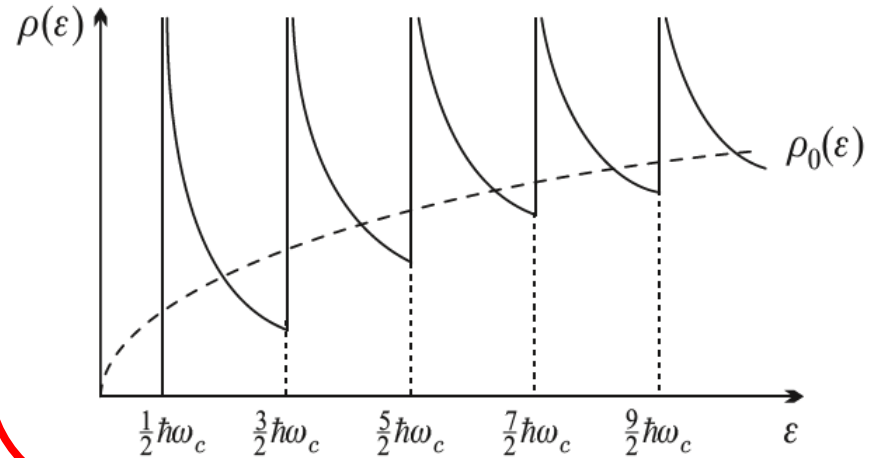


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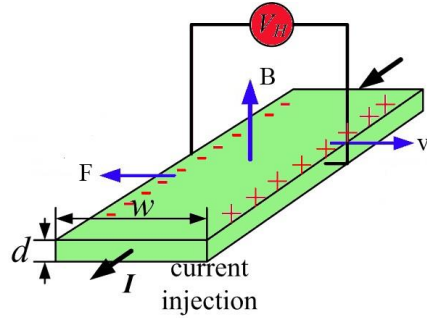


**Lista:**  $\chi_{Landau} = -\frac{1}{3}\chi_{Pauli}$

# Classical Hall effect

Revisando o efeito Hall clássico ...

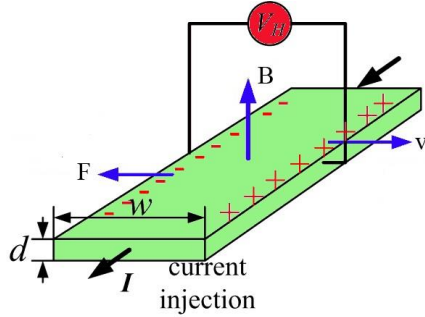
$$\frac{d\langle\vec{p}\rangle}{dt} = -e(\vec{E} + \langle\vec{v}\rangle \times \vec{B}) - \frac{\langle\vec{p}\rangle}{\tau}$$



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$$\vec{E} = \rho(B)\vec{j}$$

Lei de Ohm

$$\rho(B) = \begin{pmatrix} \frac{m}{ne^2\tau} & -\frac{B}{ne} & 0 \\ \frac{B}{ne} & \frac{m}{ne^2\tau} & 0 \\ 0 & 0 & \frac{m}{ne^2\tau} \end{pmatrix}$$

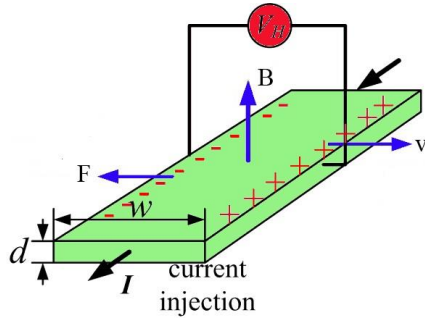
Constante Hall

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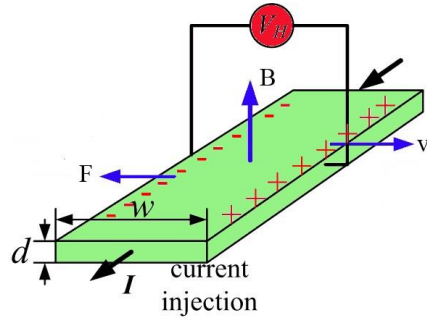
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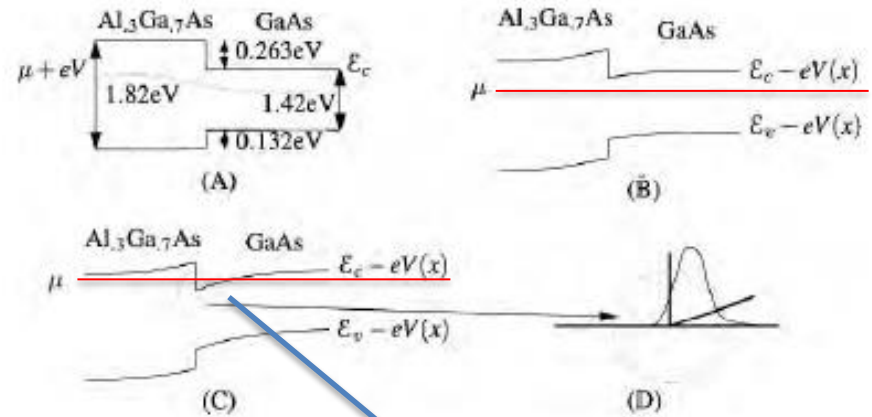
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# Quantum Hall effect

Como obter um gás de elétrons bidimensional?



2D electron gas (2DEG)

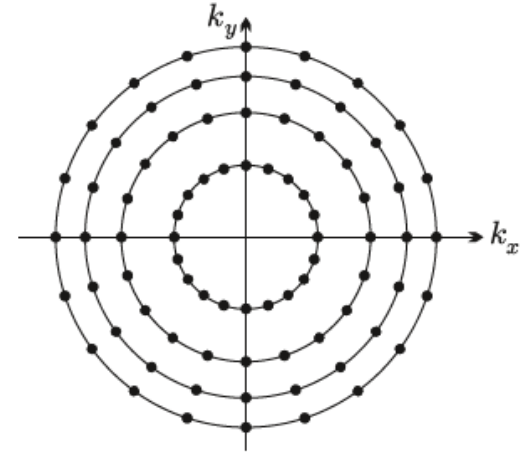
# Quantum Hall effect

Vamos supor um 2DEG sob ação de um campo magnético intenso

$$\mathcal{H} = -\frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2m_e} \left( \frac{\partial}{\partial y} + i \frac{eB}{\hbar} x \right)^2$$

$$E_n = \hbar \omega_c (n + 1/2)$$

$$\omega_c = \frac{eB}{m_e}$$



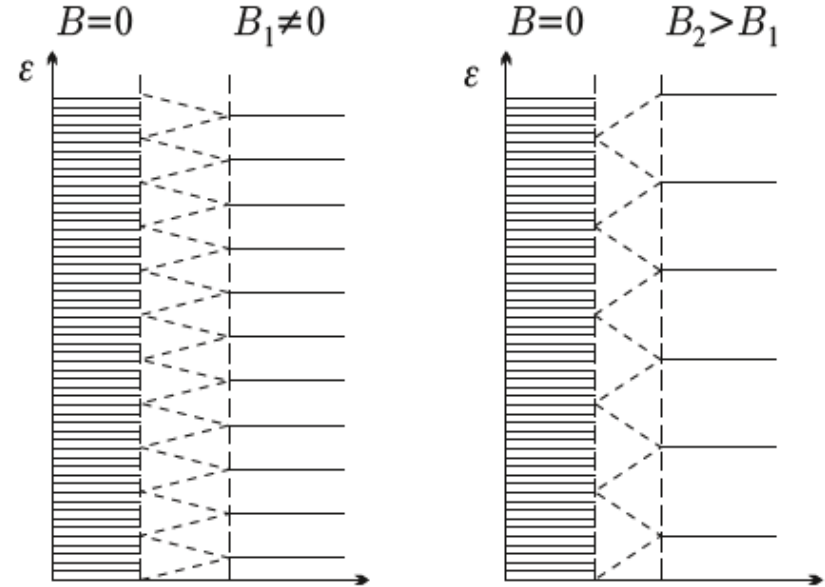
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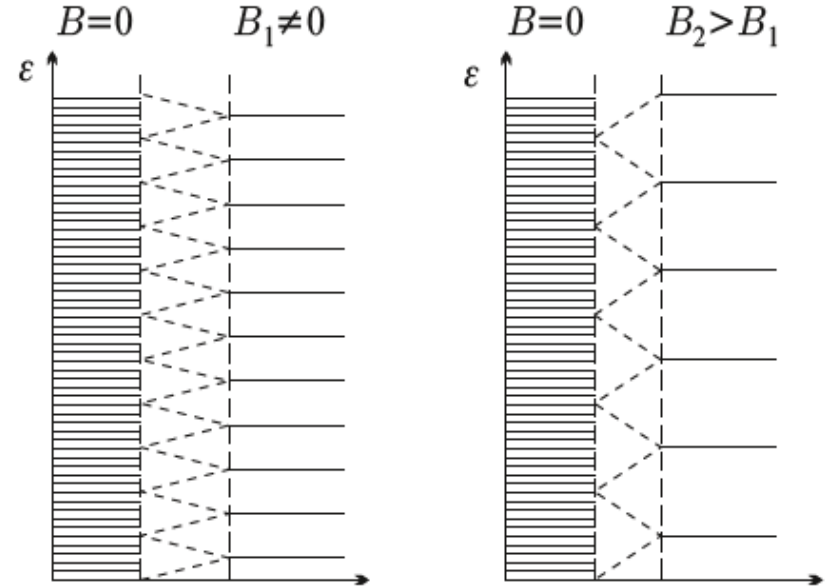
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Nível mais baixo:

$$\phi_{0,k_y} = \frac{1}{\sqrt{2\pi l^2}} e^{-(x-x_0)^2/2l^2} e^{ik_y y}$$

$$\left\{ \begin{array}{l} x_0 = -l_0^2 k_y \\ l_0 = \sqrt{\frac{\hbar}{m_e \omega_c}} = \sqrt{\frac{\hbar}{eB}} \end{array} \right.$$

# Quantum Hall effect

Degenerescência dos níveis de Landau para um 2DEG

$$0 < x_0 < L_x \Rightarrow -\frac{m_e \omega_c}{\hbar} L_x < k_y < 0$$

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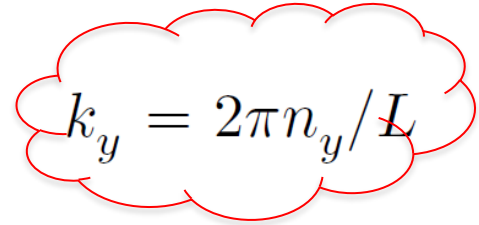
$$k_y = 2\pi n_y / L$$

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**inteiro**



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**inteiro**

$$k_y = 2\pi n_y / L$$

$$\Phi_0^* = 2\pi\hbar/e = h/e$$

Quantum de  
fluxo magnético

# Quantum Hall effect


Assumindo como correto os resultados clássicos, temos:

$$\rho_{xy} = \frac{B}{n_e e}$$

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$$\rho_{xy} = \frac{B}{n_e e}$$



$$n_e = \frac{N_e}{L_x L_y} = \nu \frac{N_p}{L_x L_y} = \nu \frac{eB}{h}$$

$\nu$  número inteiro

# Quantum Hall effect

Assumindo como correto os resultados clássicos, temos:

$$\rho_{xy} = \frac{B}{n_e e} \quad \Rightarrow \quad \rho_{xy}^{(\nu)} = \frac{1}{\nu} \frac{h}{e^2}$$


$$n_e = \frac{N_e}{L_x L_y} = \nu \frac{N_p}{L_x L_y} = \nu \frac{eB}{h}$$

$\nu$  número inteiro

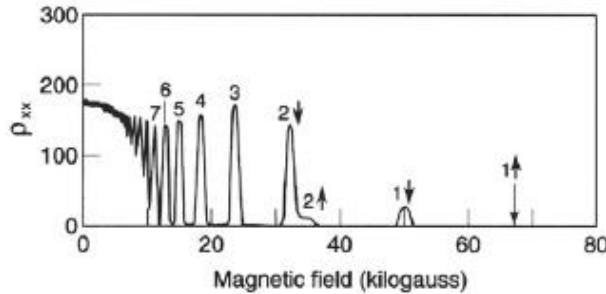
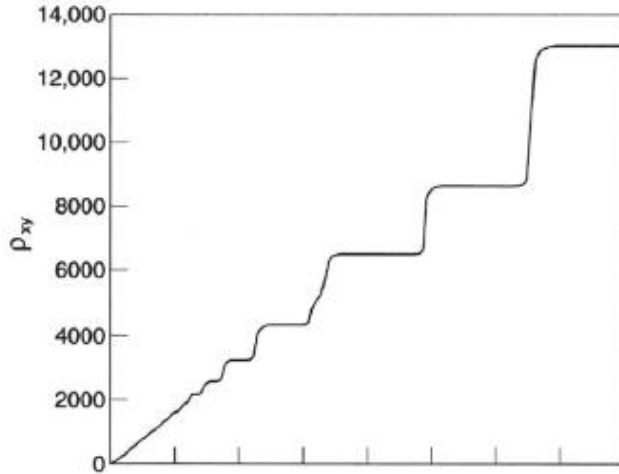
# Quantum Hall effect

Assumindo como correto os resultados clássicos, temos:

$$\rho_{xy} = \frac{B}{n_e e} \Rightarrow \rho_{xy}^{(\nu)} = \frac{1}{\nu} \frac{h}{e^2}$$

$$n_e = \frac{N_e}{L_x L_y} = \nu \frac{N_p}{L_x L_y} = \nu \frac{eB}{h}$$

$\nu$  número inteiro



$$\rho_{xy}^{(\nu)} = \frac{1}{\nu} \frac{h}{e^2}$$

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

$$R_K = h/e^2$$

$$R_K = 25.812\,807\,572 \text{ k}\Omega$$

*von Klitzing constant*

# Quantum Hall effect

